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Reduction of Data for Piston Gage Pressure Measurements

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Contents

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Pressure measurements made with piston gages are affected by gravity, temperature, pressure, and several other variables. For accurate determinations of pressure the calculations must take these variables into account. A general equation is developed and simplified procedures for calculating pressure are illustrated.

1. Introduction

The dead weight piston gage is one of the few instruments that can be used to measure pressure in terms of the fundamental units, force, and area. Piston gages are known by several names such as "dead weight gage," "dead weight tester," "gage tester," and "pressure balance." The name, "piston manometer," is used in Germany, and, since it aptly describes the instrument, might very well be used more extensively in this country. In principle, it is a piston inserted into a close fitting cylinder. Weights loaded on one end of the piston are supported by fluid pressure applied to the other end. Construction of piston gages varies as to method of loading, methods of rotating or oscillating the piston to reduce friction, and design of the piston and cylinder. Three designs of cylinders are commonly used; the simple cylinder with atmospheric pressure on the outside; the re-entrant cylinder with the test pressure applied to the outside as well as the inside; and the controlled clearance cylinder with an external jacket in which hydraulic pressure can be applied so as to vary the clearance between the piston and cylinder at the will of the operator. In order to use the piston gage for the measurement of pressure, one must take into account a number of parameters of the instrument and its environment.

Error in measurement results from failure to account for the parameters or from the uncertainty of the measured values of them. It is obvious that error results from the uncertainty of the mass of the loading weights and the measurement of the effective area of the piston and cylinder. Other sources of error may not be so easily recognized. Such sources include the air buoyancy on the weights, the fluid buoyancy on the piston, the value of local gravity, the force on the piston due to the surface tension of the fluid, the thermal expansion and elastic deformation of the piston and cylinder, and the fluid heads involved. These effects can be evaluated and corrections applied to reduce the magnitude of overall error of measurement.

Air buoyancy corrections amount to about 0.015 percent of the load. The corrections for the buoyancy of the pressure fluid on the piston have been found to range from zero to nearly 0.5 psi, and could be larger. A brief discussion of gravity error is given by Johnson and Newhall [1],[1] and detailed information can be found in the Smithsonian Meteorological Tables [2]. The values of local gravity differ by over 0.3 percent at different places in the United States. The pressure correction due to surface tension is usually negligible, but may amount to more than 0.003 psi. Thermal expansion of the effective piston area is usually about 0.003 percent per °C and elastic distortion may amount to 0.05 percent at 10,000 psi. Fluid head amounts to about 0.03 psi per inch for lubricating oils.

Other errors resulting from corkscrewing (vertical force caused by a helical scratch on the piston, cylinder, or guide bearing), vertical component of the force used to rotate or oscillate the piston, eccentric loading of the piston, and the force of air currents against the weights, are not readily evaluated. They may be kept small by good design and workmanship and by careful operating technique, but usually no attempt is made to apply corrections for them.

2. Pressure

The pressure in any system may be defined as

$$P=\frac{F}{A} \qquad (1)$$

where F=force and A=area over which the force is applied. When a piston gage is in equilibrium with a pressure system, the pressure, p_p, measured at the piston gage is

$$p_p=\frac{F_e}{A_e} \qquad (2)$$

where F_e=the force due to the load on the piston, A_e=the effective area of the piston.

[1] Figures in brackets indicate the literature references on page 6.

The pressure p_p, is not necessarily the quantity desired. It is the pressure measured by the piston gage at its reference level, whereas, the pressure that we may wish to measure may be at another level within a system which is connected to the piston gage by a length of tubing filled with a pressure transmitting fluid. The pressure exerted by the head of fluid in the connecting line, from the level within the system to the reference level of the piston gage, must be added to the piston gage pressure. If the total (absolute) pressure is to be determined, the atmospheric pressure at the reference level of the piston gage must also be added to the piston gage pressure, so we have

$$P=p_p+H_{fp}+P_a \quad (3)$$

where P=total pressure at a particular level in the system,

p_p=pressure at the reference level of the piston gage (piston gage pressure),

H_{fp}=pressure exerted by the head of pressure transmitting fluid,

P_a=atmospheric pressure at the reference level of the piston gage.

Usually the quantity to be measured is the difference between the total internal pressure in the system and the atmospheric pressure outside of the system. If the pressure is to be measured at a level in the system (which might be the reference level of another gage) which is different from the reference level of the piston gage, the atmospheric pressure at these levels is different and the equation for the difference between the internal and external pressure or gage pressure, p_g, of the system is

$$p_g=p_p+H_{fp}-H_a \quad (4)$$

where H_a=the difference in the atmospheric pressure between the reference level of the piston gage and the level in the system at which the pressure is to be measured.

3. Force and Fluid Heads

The force, F, due to the gravitational attraction between the earth and a mass, M, is numerically

$$F=kMg_L \quad (5)$$

where g_L is the local acceleration due to gravity, and k is a proportionality factor, the numerical value of which depends upon the units of F, M, and g_L as follows:

$k=1$, for F in dynes, M in grams, and g_L in cm/sec^2,

$k=1$, for F in newtons, M in kilograms, and g_L in meter/sec^2,

$k=1$, for F in poundals, M in pounds mass, and g_L in feet/sec^2,

$k=1$, for F in pounds force, M in slugs, and g_L in feet/sec^2,

$k=\frac{1}{32.174}$, for F in pounds force, M in pounds mass, and g_L in feet/sec^2,

$k=\frac{1}{980.665}$, for F in pounds force, M in pounds mass, and g_L in cm/sec^2,

$k=\frac{1}{980.665}$, for F in kilograms force (in some countries, kiloponds, kp), M in kilograms, and g_L in cm/sec^2.

There are several quantities that must be used in the determination of the force, F_e, (eq (2)) acting upon the effective area of the piston. These include, the mass of the weights, M_m, the mass of the air displaced by the load, M_a; the mass of the pressure fluid contributing to the load, M_f; the local acceleration due to gravity, g_L, and the force due to the surface tension, γ, of the pressure fluid acting upon the circumference of the piston, C, where it emerges from the surface of the fluid. The value for F_e can be determined from the following equation,

$$F_e=(M_m+M_f-M_a)kg_L+\gamma C. \quad (6)$$

3.1. Mass of Weights

The mass of the weights, including the piston and all parts which contribute to the load on the piston when in operation, is determined by comparison with the mass of accurately known standard weights. This is usually done by means of an equal arm balance.

3.2. Mass of Air

The mass of air displaced by the load on the piston is

$$M_a=\rho_a V_m, \quad (7)$$

where ρ_a=the mean density of the air displaced by the load. The volume of the load,

$$V_m=\frac{M_m}{\rho_m}+\frac{M_f}{\rho_f}, \quad (8)$$

where ρ_m=the density of the weights, M_m, and ρ_f=the density of the pressure fluid, M_f. Substituting eq (8) in eq (7) we get

$$M_a=\frac{\rho_a}{\rho_m}M_m+\frac{\rho_a}{\rho_f}M_f. \quad (9)$$

The value, ρ_m, depends upon the way in which the values for the loading weights are reported. When they are reported as true mass, the actual value for the density of the weights should be used. If, however, the apparent mass is given, as determined by comparison with brass standards in air having a density of 0.0012 g/cm³, the density of the weights should be assumed to have the same density as the brass standards (8.4 g/cm³). Apparent mass values are usually used when reporting loading weight values.

By substituting eq (9) in eq (6) we obtain the following expression for F_e,

$$F_e=\left[M_m\left(1-\frac{\rho_a}{\rho_m}\right)+M_f\left(1-\frac{\rho_a}{\rho_f}\right)\right]kg_L+\gamma C. \tag{10}$$

Piston gages operating with the piston assembly partially submerged in a liquid are subjected to a force resulting from the surface tension of the liquid acting on the periphery of the piston where it emerges from the surface of the liquid. This force, γC, is added to the force due to the load on the piston.

3.3. Fluid Head

The fluid head pressure, H_{fp}, eq (3) and (4), exerted by the column of pressure fluid between the reference level of the piston gage and the reference level of the pressure system to be measured, may be calculated as follows:

$$H_{fp}=-\rho_{fp}h_{fp}kg_L \tag{11}$$

where ρ_{fp} = the density of the pressure fluid at pressure P,

h_{fp} = the height of the column of fluid of density ρ_{fp}, measured from the reference level of the piston gage. Measurements up from the reference level are positive and down from the reference level are negative.

3.4. Air Head

The air head pressure, H_a, eq (4), exerted by the column of air between the reference level of the piston gage and the reference level of the pressure system to be measured, may be calculated as follows:

$$H_a=-\rho_a h_a kg_L \tag{12}$$

where ρ_a = the density of the air at the ambient atmospheric pressure, and temperature,

h_a = the height of the air column measured from the reference level of the piston gage. Measurements above the reference level have a positive sign and those below have a negative sign.

3.5. Fluid Buoyancy

In certain instances, the pressure fluid in which the piston is immersed contributes to the load on the piston. This effect may be accounted for in two ways. One method is to compute the mass of the fluid, M_f, contributing to the load on the piston and include it in the calculation of the force as shown in eq (6). The other method is to shift the reference level (the level at which the piston gage pressure, p_p, is measured) from the lower end of the piston by an amount equal to the height of a column of oil that will compensate for the mass of the fluid acting on the piston.

In order to determine the contribution of the pressure fluid to the load on the piston, first consider the case of a piston of uniform cross section exactly fitting the cylinder (fig. 1). If the pres-

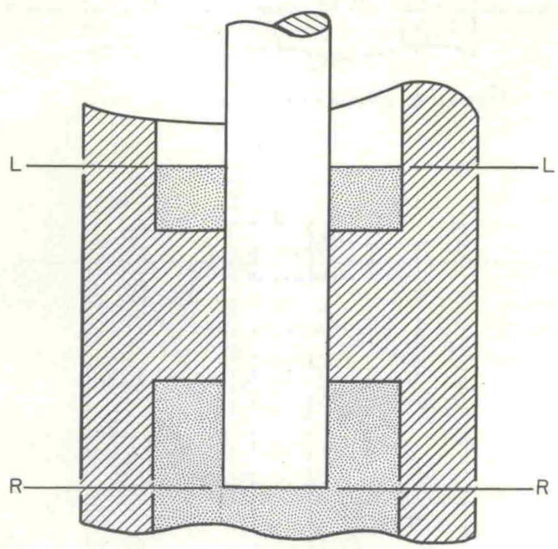

FIGURE 1. *Piston of uniform cross section*

sure is measured at the level of the lower end of the piston (level R), the fluid exerts no vertical forces other than that against the area of the end of the piston. In this case, therefore, there is no buoyancy correction to be applied.

Next consider a piston with grooves or holes machined in it as shown in figure 2. The weight of the fluid contained in the grooves and holes, is part of the load on the piston. That is to say that all of the material included within a cylinder having a cross section equal to the effective area of the piston is included in the load on the piston.

In practice, the effective area of a piston is very nearly equal to the mean of the areas of the piston and the cylinder as shown in figure 3 by the dotted lines C and C′. Any metal extending beyond those bounds displaces a volume of fluid the mass of which must be subtracted from the load on the piston and any fluid within those bounds must be added to the load on the piston.

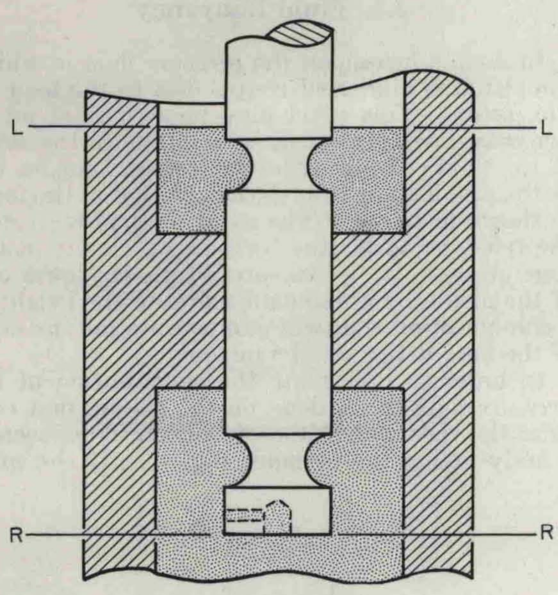

FIGURE 2. *Piston of irregular cross section.*

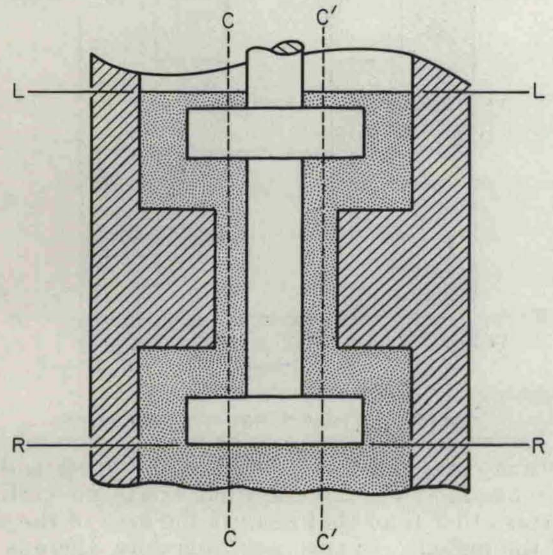

FIGURE 3. *Piston of irregular cross section.*

From these examples we see that the fluid buoyancy correction can be either positive or negative. It can be calculated from the following equation:

$$M_f=(A_e y_f-V_s)\ \rho_f \tag{13}$$

where A_e=the effective area of the piston as before,
y_f=the length of the submerged part of the piston,
V_s=the volume of the submerged part of the piston.

To determine the pressure equivalent, p_f, of the fluid, M_f, the following equation may be used:

$$p_f=\frac{M_f}{A_e}\left(1-\frac{\rho_a}{\rho_f}\right) kg_L \tag{14}$$

Shifting the reference level of the piston gage by an amount, Δh, effectively consists of adding the pressure exerted by a column of fluid of height, Δh, and density, ρ_{fp}, and subtracting the pressure exerted by a column of air of height, Δh, and density, ρ_a. The resulting pressure change,

$$\Delta p=\rho_{fp}\Delta hkg_L-\rho_a\Delta hkg_L. \tag{15}$$

By shifting the reference level an amount sufficient to make the resulting fluid head compensate for the pressure equivalent of the fluid buoyancy [3, 4] $\Delta p=p_f$ we obtain (from eqs 13, 14, and 15),

$$\Delta h=\frac{(A_e y_f-V_s)}{A_e}\frac{(\rho_f-\rho_a)}{(\rho_{fp}-\rho_a)}, \tag{16}$$

When the buoyancy correction is determined as shown in eq (13) the density of the fluid, ρ_f, is included. For the portion of the piston between the cylinder and the surface of the fluid (level L) the value of ρ_f will be the density of the fluid at atmospheric pressure, ρ_{fa}, but for the portion of the piston below the cylinder the value of ρ_f will be the density of the fluid, ρ_{fp}, at pressure, P, and may not be easily determined.

On the other hand, when the buoyancy correction for the lower end of the piston is applied by shifting the reference level, ρ_f in eq (16) is equal to ρ_{fp}, and the ratio $\frac{(\rho_f-\rho_a)}{(\rho_{fp}-\rho_a)}$ is equal to 1, so that the value for ρ_{fp} need not be known.

3.6. Mass of Fluid

By applying the buoyancy correction for the upper portion of the piston as a load correction, eq (13) becomes

$$M_{fa}=(A_e y_{fa}-V_{fa})\rho_{fa} \tag{17}$$

where y_{fa}=the length of the submerged part of the piston above the cylinder,
V_{fa}=the volume of the submerged part of the piston above the cylinder,

and eq (10) becomes

$$F_e=M_m\left[\left(1-\frac{\rho_a}{\rho_m}\right)+M_{fa}\left(1-\frac{\rho_a}{\rho_{fa}}\right)\right]kg_L+\gamma C. \tag{18}$$

3.7. Reference Level

By applying the buoyancy correction for the lower portion of the piston as a reference level change, eq (16) becomes

$$\Delta h = y_{fp} - \frac{V_{fp}}{A_e} \quad (19)$$

where y_{fp}=the length of the piston below the cylinder,
V_{fp}=the volume of the piston below the cylinder.

4. Area

The effective area, A_e, of the piston, can be expressed by the relationship

$$A_e = A_0[1+a(t-t_s)](1+bp_p)[1+d(p_z-p_j)] \quad (20)$$

where A_0=the effective area of the piston at temperature, t_s, and atmospheric pressure,
a=the fractional change in effective area per unit change in temperature,
t=the temperature of the piston and cylinder,
t_s=the reference temperature,
b=the fractional change in effective area per unit change in pressure,
d=the fractional change in effective area per unit change in jacket pressure of a controlled clearance piston gage,
p_z=the jacket pressure required to reduce the piston clearance to zero at pressure p_p,
p_j=the jacket pressure.

4.1. Temperature Coefficient of Area

The fractional change in effective area per unit change in temperature can be determined as follows:

$$a = \alpha_k + \alpha_c \quad (21)$$

where α_k=the temperature coefficient of linear expansion of the piston,
α_c=the temperature coefficient of linear expansion of the cylinder.

The most convenient reference temperature, t_s, is the average temperature of the room in which the instrument is used. In many instances the difference $t-t_s$ may be insignificant.

The temperature, t, of the piston and cylinder is usually assumed to be the same as the temperature of the base of the instrument. The fact is, in practice, the piston and cylinder are usually at a temperature higher than that of the rest of the instrument, although, in a gas lubricated piston gage they may be lower. So many factors affect the temperature that the order of magnitude calculation of the temperature rise may be unreliable. Some of the more important factors are: speed of rotation, clearance, pressure, viscosity, Joule-Thompson coefficient, and thermal conductivity of the pressure fluid. One precaution that can be taken to keep the uncertainty of temperature, t, from being unnecessarily large, is to keep the speed of rotation no greater than is required to maintain hydrodynamic lubrication.

4.2. Effective Area at Atmospheric Pressure

A_0 is very nearly equal to the mean of the area of the piston and the area of the cylinder [3,4] at the reference temperature and can be calculated as follows:

$$A_0 = \frac{A_k + A_c}{2}[1+a(t_s-t_m)] \quad (22)$$

where A_k=the area of the piston,
A_c=the area of the cylinder,
t_m=the temperature at which A_k and A_c are measured.

The value of $\frac{A_k+A_c}{2}$ may be determined from direct measurements of the diameters of the piston and cylinder, or by comparison with a piston gage of known area. In a controlled clearance piston gage, the jacket pressure, applied to the outside of the cylinder, is used to vary the diameter of the cylinder as desired and A_c is assumed to be equal to A_k.

4.3. Pressure Coefficient of Area

The fractional change in area with pressure is most readily obtained by comparing the instrument against a controlled clearance piston gage for which the pressure coefficient, b, can be computed, or by comparison with a piston gage of known characteristic. To the first approximation, for a controlled clearance piston gage,

$$b = \frac{3\mu - 1}{Y} \quad (23)$$

where μ=Poisson's ratio for the piston,
Y=Young's modulus for the piston.

The product, bp_p, is small and therefore an approximate value for p_p is adequate. Deffet and Trappeniers [4], Dadson [6], Bridgman [7], and Ebert [8] give more detailed discussions of the elastic distortion of piston gages.

4.4. Change of Area With Jacket Pressure

The factor $[1+d(p_z-p_j)]$ is used only for controlled clearance piston gages. The fractional change of area with jacket pressure, d, is best obtained experimentally by varying the jacket pressure, p_j, and measuring the change in pressure, p_p. Since the change in pressure will be small, the measuring instrument must be sensitive.

The jacket pressure required to reduce the clearance between the piston and cylinder to zero, at a particular value of p_p, may be determined by two methods. One method is to observe the torque required to turn the piston as the jacket pressure is varied. An abrupt increase of torque is observed when the clearance is reduced to zero. The other method is to measure the fall rate of the piston at several jacket pressures. The cube root of the fall rate is then plotted against jacket pressure, and the curves are extrapolated to zero fall rate to get values of p_z.

5. Piston Gage Pressure

The complete equation for p_p can be obtained from combining eqs (2), (18), and (20);

$$p_p=\frac{\frac{M_m}{A_o}\left(1-\frac{\rho_a}{\rho_m}\right)kg_L+\frac{M_{fa}}{A_o}\left(1-\frac{\rho_a}{\rho_{fa}}\right)kg_L+\frac{\gamma C}{A_o}}{[1+a(t-t_s)](1+bp_p)[1+d(p_z-p_j)]} \tag{24}$$

where the reference level is determined as shown in eq (19).

Equation (24) would be rather formidable if it were necessary to solve it for each pressure measurement. Fortunately the terms can be grouped so that the amount of calculation can be reduced to practical proportions for some instruments, and some terms can be ignored if the accuracy requirements are low. It should be noted that eq (24) is not exact. Some second order terms have been dropped and the coefficients are constant only to a first approximation.

6. Conclusions

The accuracy of pressure measurements depends not only on the performance of the piston gage, but on the application of corrections derived from parameters of the system. These depend upon the construction of the instrument, composition of the pressure fluid, environment, pressure, and physical arrangement of the pressure system. The accuracy to which the values of these parameters are known usually establishes the overall accuracy of the measurements.

7. References

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8. Appendix A. Computation of Pressure

8.1. Calculation of Correction Factors

To illustrate a method for simplifying the calculations, suppose that a particular piston gage is to be used in Room 131, MTL Bldg. at the National Bureau of Standards in Washington, D.C., to calibrate Bourdon gages. The local value of gravity g_L=980.10 gals, ρ_a will be assumed to have the value of the average density of air at this location, so $\rho_a=0.00117$ g/cm^3. The fluid being used in the instrument is aviation instrument oil for which the density, $\rho_{fa}=0.862$ g/cm^3 or 0.0321 lb/in.^{3} and the surface tension, $\gamma=0.00017$ lbf/in., at atmospheric pressure and 25 °C.

The piston gage is not a controlled clearance piston gage, therefore the factor $1+d(p_z-p_j)$ will be omitted. From direct measurements on the instrument, we find that $C=1.964$ in., $y_{fa}=2.5$ in., $V_{fa}=1.525$ in.^{3}, $y_{fp}=1.625$ in., and $V_{fp}=0.2778$ in.^{3}.

From a previous calibration against a controlled clearance piston gage, we have: $A_0=0.13024$ in.^{2} (at $t_s=25$ °C) and $b=1.48\times10^{-7}$ in.^{2}/in.^{2} psi. We also know that the piston is steel with the temperature coefficient, $\alpha_k=12\times10^{-6}$ in./in. deg C at 25 °C and the cylinder is brass with the temperature coefficient $\alpha_c=18.4\times10^{-6}$ in./in. degree C at 25 °C.

The masses of the loading weights have been determined and have been reported in tabular form as shown in columns 1 and 2 of table 1. The value given in column 2 is the apparent mass and therefore the density, ρ_m, is assumed to be 8.4 g/cm^3.

All of the quantities thus determined can be listed as illustrated in table 2 and from these values the factors and terms of the numerator of eq (24) and the reference level, Δh, can be computed as shown in table 2.

The importance of the various factors and terms of the numerator can now be evaluated. In the first term $\frac{M_m}{A_0}$ is the dominant factor, while the factor kg_L accounts for about 0.06 percent and the factor $\left(1-\frac{\rho_a}{\rho_m}\right)$ accounts for about 0.014 percent. The contribution of the second term is -0.295 psi with the factor $\left(1-\frac{\rho_a}{\rho_{fa}}\right)$ having an effect amounting to about 0.0004 psi. In this case the third term amounts to 0.0027 psi. The importance of each factor and term depends upon the design of the instrument, the environment, and the application, and none should be neglected without first evaluating it to determine its significance.

8.2. Machine Computation

The temperature and pressure factors, $1+a(t-t_s)$ and $(1+bp_p)$, in the denominator of eq (24) can be combined in a double entry table to give values of

$$\frac{1}{[1+a(t-t_s)](1+bp_p)}$$

for values of t and p_p as illustrated in table 3.

To further facilitate computations the weight table, table 1, has been extended to give values of M_m times the factor 7.6726 (from table 2) for each weight as shown in column (3) and the value of

$$M_{fa}\left(1-\frac{\rho_a}{\rho_{fa}}\right)\frac{kg_L}{A_0}+\frac{\gamma C}{A_0}=-0.292$$

is added to the value of 7.6726 M_m for the piston. Accumulative totals, for frequently used combinations of weights, are given in column (4). Computation of pressure p_p is now simplified to the process of multiplying the sum of values from column (3) or a value from column (4) by the appropriate value from table 3. Greater simplification to suit the requirements of specific applications will be left to the ingenuity of the user.

8.3. Slide Rule Computation

The method just described can be modified slightly for use with a slide rule. Using the same values as were used in the preceding illustration, eq (24) for p_p may be written in the form

$$p_p=(M_m 7.6726-0.292)+(M_m 7.6726-0.292)\times\left(\frac{1}{[1+a(t-t_s)](1+bp_p)}-1\right). \quad (25)$$

TABLE 1. *List of weights and appropriate values for weight set and piston gage No. 1357 used at Washington, D.C., with aviation instrument oil*

Piston gage No. 1357 with weight set No. 1357
Location—NBS, Washington, D.C.
Fluid—Aviation instrument oil

(1) Weight No.	(2) Mass (M_m)	(3) $M_m\times 7.6726$	(4) Accumulative Total
	lb	*psi*	*psi*
piston	1.3024	9.993−0.292* =9.701	9.701
1	2.6042	19.981	29.682
2	2.6045	19.983	49.665
3	6.5123	49.966	99.631
4	26.0473	199.85	299.48
5	26.0454	199.84	499.32
6	65.1095	499.56	998.88
7	65.1190	499.63	1498.51
8	65.1065	499.54	1998.0
9	65.1165	499.61	2497.7

*Fluid buoyancy and surface tension correction,

$$M_{fa}\left(1-\frac{\rho_a}{\rho_{fa}}\right)\frac{kg_L}{A_0}+\frac{\gamma C}{A_0}=-0.292 \text{ psi}.$$

TABLE 2. *Tabulated values of parameters for piston gage No. 1357 used at Washington, D.C., with aviation instrument oil*

Values for Piston Gage No. 1357

$A_0=0.13024$ in^2.
$y_{fa}=2.5$ in.
$V_{fa}=1.525$ in^3.
$C=1.964$ in.
$a=\alpha_k+\alpha_c=(12+18.4)\times 10^{-6}=30.4\times 10^{-6}$ in^2./in^2. °C
$t_s=25$ °C
$b=1.48\times 10^{-7}$ in^2./in^2.psi
$y_{fp}=1.625$ in.
$V_{fp}=0.2778$ in^3.

Values for Weight Set No. 1357

$\rho_m=8.4$ g/cm^3

Values for Room 131, MTL Bldg., NBS, Washington, D.C.

$\rho_a=0.00117$ g/cm^3 or 0.0000423 lb/in^3.
$g_L=980.10$ cm/sec^2

Values for Aviation Instrument Oil

$\rho_{fa}=0.0321$ lb/in^3.
$\gamma=0.00018$ lbf/in.

Computations

$kg_L=0.99942$

$\frac{kg_L}{A_0}=7.6737$/in^2.

$\left(1-\frac{\rho_a}{\rho_m}\right)=0.99986$

$M_m\left(1-\frac{\rho_a}{\rho_m}\right)\frac{kg_L}{A_0}=M_m\times 7.6726$ psi

$M_{fa}=(A_e y_{fa}-V_{fa})\rho_{fa}=-0.0385$ lb (17)

$\left(1-\frac{\rho_a}{\rho_{fa}}\right)=-0.9987$

$M_{fa}\left(1-\frac{\rho_a}{\rho_{fa}}\right)\frac{kg_L}{A_0}=-0.295$ psi

$\frac{\gamma C}{A_0}=0.0027$ psi

$p_p=(M_m\times 7.6726-0.292)\frac{1}{[1+a(t-t_s)](1+bp_p)}$ (25)

$\Delta h=y_{fp}-\frac{V_{fp}}{A_e}=-0.508$ in. (19)

Table 3. *Temperature and pressure correction factors for piston gage No. 1357, for calculation of pressure, in psi*

Values of $\frac{1}{[1+a(t-t_s)](1+bp_p)}$ for piston gage No. 1357					
Pressure, p_p	Temperature, t, °C				
	23	24	25	26	27
psi					
2500	0.99969	0.99966	0.99963	0.99960	0.99957
2000	.99976	.99973	.99970	.99967	.99964
1500	.99984	.99981	.99978	.99975	.99972
1000	.99991	.99988	.99985	.99982	.99979
500	.99999	.99996	.99993	.99990	.99987
0	1.00006	1.00003	1.00000	.99997	.99994

Table 4. *Temperature and pressure correction factors for piston gage No. 1357, for calculation of corrections, in psi*

Values of $\frac{1}{[1+a(t-t_s)](1+bp_p)}-1$ for piston gage No. 1357					
Pressure, p_p	Temperature, t, °C				
	23	24	25	26	27
psi					
2500	−0.00031	−0.00034	−0.00037	−0.00040	−0.00043
2000	−.00024	−.00027	−.00030	−.00033	−.00036
1500	−.00016	−.00019	−.00022	−.00025	−.00028
1000	−.00009	−.00012	−.00015	−.00018	−.00021
500	−.00001	−.00004	−.00007	−.00010	−.00013
0	+.00006	+.00003	.00000	−.00003	−.00006

A double entry table for values of

$$\frac{1}{[1+a(t-t_s)](1+bp_p)}-1$$

for various values of t and p_p can be prepared as illustrated in table 4. A slide rule can be used to multiply the appropriate value from this table by the sum of values from column (3) or a value from column (4) of table 1 to obtain a correction to be added to the value from table 1 to give pressure p_p.

Table 5. *Temperature and pressure corrections in psi, for piston gage No. 1357*

Values of $p_p\left[\frac{1}{[1+a(t-t_s)]\,(1+bp_p)}-1\right]$ for piston gage No. 1357					
Pressure, p_p	Temperature, t, °C				
	23	24	25	26	27
	Corrections, psi				
psi					
2500	−0.8	−0.8	−0.9	−1.0	−1.1
2000	−.5	−.5	−.6	−.7	−.7
1500	−.24	−.28	−.33	−.38	−.42
1000	−.09	−.12	−.15	−.18	−.21
500	.00	−.02	−.04	−.05	−.06
0	.00	.00	.00	.00	.00

8.4. Correction Table Computation

A correction table may be preferred in many instances. Again using the same values as before to illustrate, a double entry table of values of

$$p_p\left(\frac{1}{[1+a(t-t_s)](1+bp_p)}-1\right)$$

for various values of p_p and t is prepared as illustrated by table 5. Appropriate corrections from the table are added to values of M_m 7.6726−0.292 obtained from table 1 to obtain pressure p_p.

8.5. Conclusions

By construction of tables a procedure similar to one of those illustrated can be established to suit the particular needs and application of the user. The computation of pressure from piston gage data is thereby reduced to a simple, fast operation.

9. Appendix B. Working Equations

The equations that may be required for the computation of the absolute or the gage pressure in a system, from measurements made with a piston gage, are listed below:

$$P=p_p+H_{fp}+P_a \tag{3}$$

$$p_g=p_p+H_{fp}-H_a \tag{4}$$

$$H_{fp}=-\rho_{fp}h_{fp}kg_L \tag{11}$$

$$H_a=-\rho_a h_a kg_L \tag{12}$$

$$A_0=\frac{A_k+A_c}{2}[1+a(t_s-t_m)] \tag{22}$$

$$a=\alpha_k+\alpha_c \tag{21}$$

$$b=\frac{3\mu-1}{Y} \tag{23}$$

$$M_{fa}=(A_e y_{fa}-V_{fa})\rho_{fa} \tag{17}$$

$$\Delta h=y_{fp}-\frac{V_{fp}}{A_e} \tag{19}$$

$$p_p=\frac{\frac{M_m}{A_0}\left(1-\frac{\rho_a}{\rho_m}\right)kg_L+\frac{M_{fa}}{A_0}\left(1-\frac{\rho_a}{\rho_{fa}}\right)kg_L+\frac{\gamma C}{A_0}}{[1+a(t-t_s)](1+bp_p)[1+d(p_z-p_j)]} \tag{24}$$

Definitions of the Symbols Used in the Above Equations

A_c Cylinder area.
A_e Effective area of piston.
A_k Piston area.
A_0 Effective area of the piston at atmospheric pressure and temperature t_s.
C Circumference of the piston at the surface of the pressure fluid.
H_a Pressure difference in the atmosphere between the reference level of the piston gage and the reference level of the system to be measured.
H_{fp} Pressure head of the column of pressure transmitting fluid between the reference level of the piston gage and the reference level of the system to be measured.
M_{fa} Mass of the pressure fluid at atmospheric pressure contributing to the load on the piston.
M_m Mass of the loading weights, including the piston assembly.
P Absolute (total) pressure.
P_a Atmospheric pressure at the reference level of the piston gage.
V_{fa} Volume of the submerged part of the piston above the cylinder.
V_{fp} Volume of the part of the piston below the cylinder.
Y Young's modulus.
a Fractional change in effective area with unit change in temperature.
b Fractional change in effective area with unit change in pressure.
d Fractional change in area with unit change in jacket pressure.
g_L Local acceleration due to gravity.
h_a Height of the air column measured from the reference level of the piston gage to the reference level of the system. Measurements up from the piston gage reference level are positive.
h_{fp} Height of the column of pressure fluid measured from the reference level of the piston gage to the reference level of the system. Measurements up from the piston gage reference level are positive.
Δh Height of the reference level of the piston gage with respect to the bottom of the piston. Measurements up from the bottom of the piston are positive.
k Proportionality factor relating force, mass and gravity.
p_g Gage pressure.
p_j Jacket pressure.
p_p Pressure measured by piston gage at the reference level of the piston gage.
P_z Jacket pressure required to reduce the piston-cylinder clearance to zero.
t Temperature of the piston gage.
t_m Temperature at which piston and cylinder are measured.
t_s Reference temperature (usually the nominal room temperature).
y_{fa} Length of the submerged part of the piston above the cylinder.
y_{fp} Length of the part of the piston below the cylinder.
α_c Temperature coefficient of linear expansion of the cylinder.
α_k Temperature coefficient of linear expansion of the piston.
γ Surface tension of the pressure fluid.
μ Poisson's ratio for the piston.
ρ_a Mean density of the air displaced by the load.
ρ_{fa} Density of the pressure fluid at atmospheric pressure.
ρ_{fp} Density of the pressure fluid at pressure P.
ρ_m Density of the weights.

10. Appendix C. Examples of Calculations

Fluid—Aviation instrument oil
Piston gage No. 1357, Washington, D.C.
Machine Calculation:

a. Weights: Piston, 1, 2, 3, 4, 5, 6, 7, 8
Accumulative total: 1998.0 psi (from table 1, column (4))
Temperature: 26 °C
Correction factor: 0.99967 (from table 3)
$p_p=1998.0\times0.99967=\underline{\underline{1997.3}}$ psi

b. Weight No. $M_m\times7.6726$ (from table 1, column (3))

Piston	9.701	99.631 accumulative total from column (4) (Piston–3)
1	19.981	
2	19.983	
3	49.966	
6	499.56	
7	499.63	
8	499.54	
	1598.361 psi	

Temperature: 26 °C
Correction factor: 0.99975 (from table 3)
$p_p=1598.36\times0.99975=\underline{\underline{1598.0}}$ psi

Slide Rule Calculation:

a. Weights: Piston, 1, 2, 3, 4, 5, 6, 7, 8
Accumulative total: 1998.0 psi (from table 1, column (4))
Temperature: 26 °C
Correction factor: −0.00033 (from table 4)
Correction = −0.00033×1998.0 = −0.7 psi
$p_p=1998.0-0.7=\underline{\underline{1997.3}}$ psi

Correction Table Calculation:

a. Weights: Piston, 1, 2, 3, 4, 5, 6, 7, 8
Accumulative total: 1998.0 (from table 1, column (4))
Temperature: 26 °C
Correction = −0.7 psi (from table 5)
$p_p=1998.0-0.7=\underline{\underline{1997.3}}$ psi

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